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# Critical Velocity of Electromagnetic Rail Gun in Response to Projectile Movement

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# **Army Research Laboratory**

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## **Critical Velocity of Electromagnetic Rail Gun in Response to Projectile Movement**

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Weapons and Materials Research Directorate, ARL

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## **Abstract**

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A model is developed to investigate the dynamic response of an electromagnetic rail gun induced by a moving magnetic pressure during launch of projectiles. As the projectile velocity approaches a critical value, resonance can occur and cause high-amplitude stress and strain in the rail at the instant and location of the projectile's passage. In this study, governing equations of a railgun under dynamic-loading conditions are derived that illustrate a lower-bound critical velocity in terms of material properties, geometry, and barrel cross section. A study is then performed to show the effect of these parameters on the critical velocity of the barrel. Accordingly, the model that accounts for projectile velocity and gun construction can be used to guide barrel design.

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## 1. Introduction

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A strain of very high amplitude and frequency, commonly referred to as dynamic strain amplification, develops in a conventional gun tube at the passage of the projectile. The phenomenon is caused by the resonance of flexural waves when the moving pressure approaches the velocity of wave propagation in the gun tube. The resonance response in an isotropic cylinder attributed to a moving pressure load has been investigated by Taylor [1], Jones and Bhuta [2], Tang [3], and Reismann [4]. Simkins [5] investigated the dynamic response of flexural waves in steel gun tubes, as very large strains have been observed in a 120-mm tank gun barrel. Hopkins [6] applied finite-element analysis to obtain a solution in a more complex taper geometry. Tzeng and Hopkins [7] investigated the dynamic strain effect in cylinders made of fiber-reinforced composite materials overwrap with a metal liner. Tzeng [8] extended the research to study fracture in the composite gun tube due to the dynamic response.

In this report, an analytical solution was developed to obtain the critical velocity of an electromagnetic (EM) rail gun barrel attributed to dynamic-loading conditions. Dynamic response could be a concern, particularly when a fieldable EM barrel has to be of lightweight construction with a hypervelocity launch capability. Figure 1 shows a schematic of an EM rail gun cross section and loading condition [9]. The rail and insulator (typically ceramic or polymer composite) were contained and supported by a containment structure. The rails are in compression due to the EM force acting on the rail and reaction force resulting from the containment structure. Furthermore, the magnetic force in the rail is discontinuous at the location of projectile armature where the electrical current passes through. The discontinuity of the force causes a local-bending moment and shear stress in the rails near the armature location. The pressure front will move along the rails as the projectile moves down through the barrel. Accordingly, dynamic stress and strain occur as the projectile movement approaches the critical velocity of the railgun.

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## 2. Analysis

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Consider a railgun cross section, as shown in Figure 1. The rail has a rectangular cross section and is mechanically supported by a rigid insulation and a containment structure [10]. The structural response of the rail can be modeled as a beam sitting on an elastic foundation, as shown in Figure 2. Accordingly, the rail is the beam and the support from the insulation material and containment is modeled as an elastic foundation. It is assumed that structural interaction between the rail and the containment is modeled through the elastic constant. The magnetic

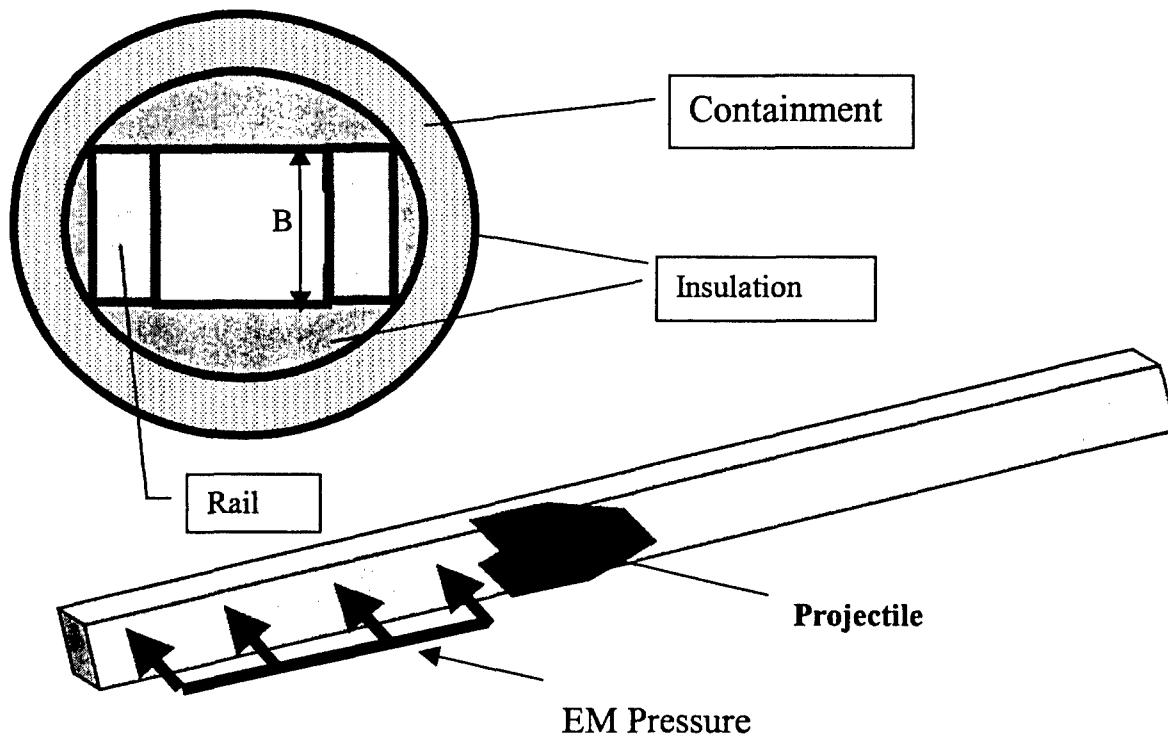


Figure 1. A schematic of the EM gun cross section, rail, and loading conditions.

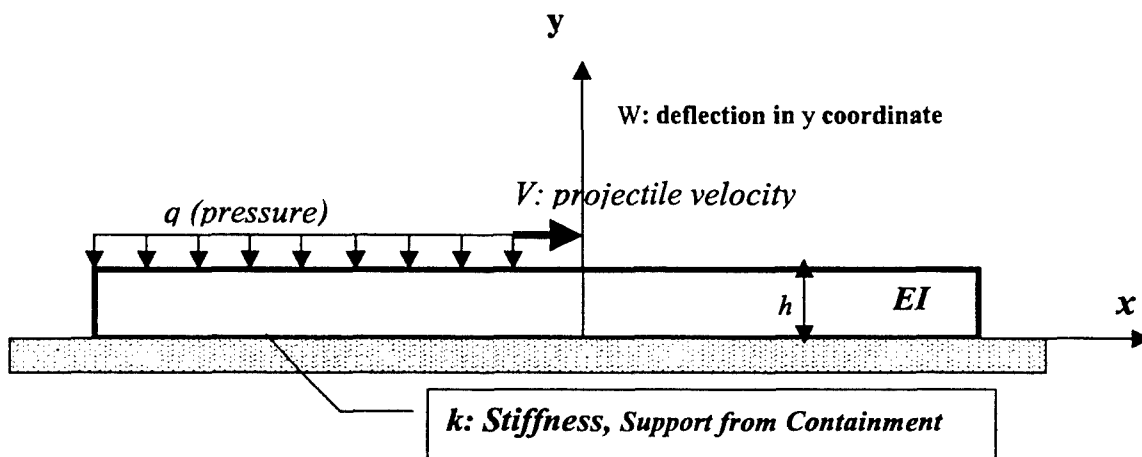


Figure 2. Coordinates system and model simulation.

pressure traveling at the speed of the projectile on the rail can be expressed as a Heaviside step function. The governing equation for the rail gun subjected to a moving pressure can then be derived as follows:

$$m \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + kw = q[1 - H(x - Vt)]. \quad (1)$$

Here,  $w$  is the lateral displacement, dependent upon time,  $t$ , and axial position coordinate,  $x$ ;  $m$  is the mass per unit length and is equal to  $\rho Bh$ ;  $\rho$  is the density of rail material; and  $B$  and  $h$  are the width and thickness of the rail, respectively.  $E$  is the modulus of rail material, and  $I$  is the moment of inertia of the rail cross section. The elastic constant,  $k$ , due to the elastic foundation, will be derived in a later section. The loading function,  $q[1 - H(x - Vt)]$  in equation (1), represents the magnetic pressure front traveling along the rail with a constant velocity,  $V$ , represented by a Heaviside step function,  $H(x - Vt)$ . The magnetic pressure,  $q$ , is assumed to be constant also. Accordingly,

$$\begin{aligned} q(1 - H(x - Vt)) &= 0 & \text{when } x > Vt \\ &= q & \text{when } x \leq Vt. \end{aligned} \quad (2)$$

Equation (1) can be solved using separation of variables with the assumption of

$$w(x, t) = \phi(t)\theta(x). \quad (3)$$

Accordingly, the left-hand side of equation (1) can be rewritten to solve the homogeneous solution as follows:

$$m\ddot{\phi}\theta + EI\phi\theta'''' + k\phi\theta = 0. \quad (4)$$

The critical velocity of the beam (rail) can be derived from the characteristic function from equation (4). The velocity spectrum can be obtained by assuming

$$w(x, t) = Ae^{iN(x - Vt)}, \quad (5)$$

where  $N$  is wave number and  $V$  is the phase velocity.

Accordingly, the critical velocity can be obtained as follows:

$$V_{cr}^2 = \frac{1}{\sqrt{3}} \frac{1}{\rho} \sqrt{\frac{h}{B}} \sqrt{EI} \sqrt{k}. \quad (6)$$

Equation (6) shows that the critical velocity of a railgun subjected to a moving pressure front is a function of the rail geometry, density, and elastic modulus. In addition, the support from the containment structure has great influence on the dynamic behavior of the rail. Critical velocity increases with the elastic modulus of the rails and the stiffness of containment structures. From a

design point of view, a launcher constructed with stiff and lightweight materials is in favor for dynamic loading conditions.

The elastic constant of foundation,  $k$ , can be calculated from the containment structure if the coupling effects of the insulation material (ceramic in general) are neglected. We consider a circular containment of a unit length subjected to concentrated loads at the inner surface of a cylinder, as shown in Figure 3(a). Accordingly, the concentrated loads are calculated from the summation of resulting magnetic pressure. Because both the containment geometry and loading conditions are symmetrical, the structural response can be calculated from the free body shown in Figure 3(b). The stiffness of the containment at the location of the concentrated load can then be obtained from the strain energy of the curved beam shown in Figure 3(b). Neglecting the shear contribution, the strain energy can be expressed as follows:

$$U = \int_0^{\pi/2} \frac{N^2 R}{2A_c E_c} d\theta + \int_0^{\pi/2} \frac{M_a^2 R}{2I_c E_c} d\theta, \quad (7)$$

where  $N$  is the normal force,  $M_a$  is the moment resulting from the concentrated load, and  $R$  is the mean radius of the containment.  $A_c$ ,  $I_c$ , and  $E_c$  are the cross-sectional area, the moment of inertia, and elastic modulus of containment, respectively.  $N$  and  $M_a$  can then be derived as follows:

$$N = \frac{P}{2} \cos \theta, \quad (8)$$

and

$$M_a = \frac{PR}{2} \left( \cos \theta - \frac{2}{\pi} \right). \quad (9)$$

Therefore, the strain energy of the quarter containment can be calculated in terms of concentrated load,  $P$ , material properties, and geometry. Based on Castigliano's theorem, the displacement at the location of the loading  $P$  can then be derived from the derivative of the strain energy with respect to the  $P$  as follows:

$$\delta_p = \frac{\partial U}{\partial P} = \frac{\pi PR}{16A_c E_c} + \frac{\pi PR^3}{16I_c E_c} \left( 1 - \frac{8}{\pi^2} \right), \quad (10)$$

where  $I_c$  is the bending moment of inertia calculated from a unit length of containment (curved beam) shown in Figure 3, which is equal to  $\frac{1}{12} bt^3$  ( $b=1$ ).  $E_c$  is Young's modulus of the containment. The stiffness constant of the containment can then be defined as

$$k = \frac{1}{\delta_p} \text{ and } P = 1. \quad (11)$$

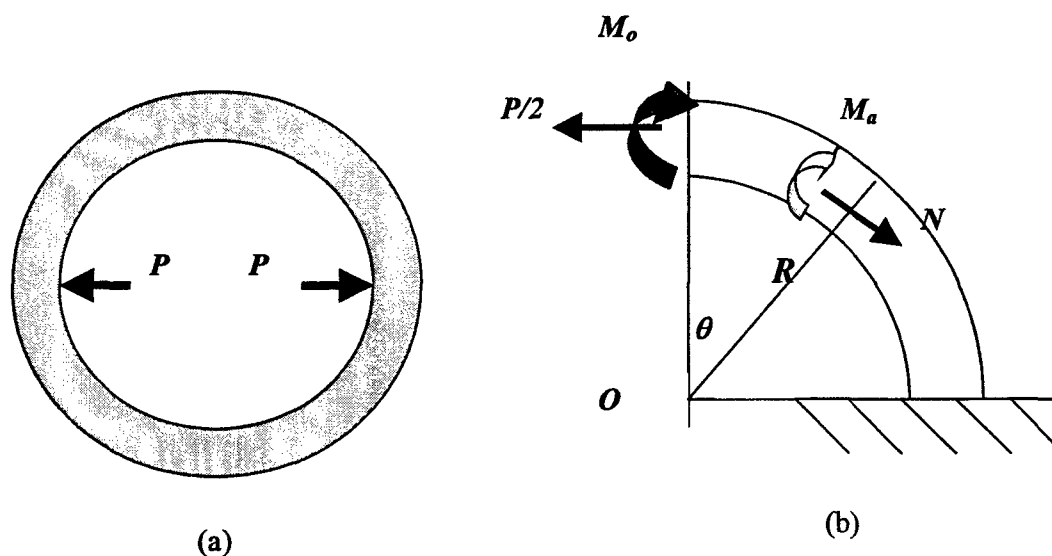


Figure 3. Modeling of containment stiffness.

### 3. Numerical Results

A baseline test case is used to obtain critical velocity for a parametric study. The gun is composed of a pair of aluminum rails, ceramic insulation, and a steel containment. The aluminum rail has a cross section of 12.5 mm (0.5 in) thick  $\times$  76.2 mm (3 in) high. The containment is 12.5 mm (0.5 in) thick. The parameters required for the simulation are listed in Table 1.

Table 1. Mechanical properties and geometric parameters of the rail and containment.

Rail (Aluminum)			
Modulus ( $E$ )	Thickness ( $h$ )	Height ( $B$ )	Density ( $\rho$ )
68.95 GPa	$12.5 \times 10^{-3}$ m	$76.2 \times 10^{-3}$ m	2750 kg/m <sup>3</sup>
Containment (Steel)			
Modulus ( $E_c$ )	Thickness ( $t$ )	Mean Radius ( $R$ )	
206.85 GPa	$12.5 \times 10^{-3}$ m	$63.5 \times 10^{-3}$ m	

The stiffness of the containment can be obtained from equations (10) and (11) by application of a unit-concentrated pressure load. The inverse of deflection at the center of the rail ( $\delta_p$ ) yields the stiffness,  $k$ , as follows:

$$k = 1 / \delta_p = 3.65 \times 10^9 \text{ Pa} . \quad (12)$$

The  $k$  represents the spring constant of foundation for the entire rail height of 76.2 mm (3.0 in). The critical velocity of the barrel can then be calculated from equation (6) as follows:

$$V_{cr} = 1148 \text{ m/s} . \quad (13)$$

The critical velocity is strongly dependent on the cross-section geometry and the mechanical properties of rail and containment. The model is based on the assumption of no structural coupling effects from insulation materials. It is a reasonable assumption because the bonding between rail and insulation is friction. Parametric studies are performed to compare the baseline case that is constructed with aluminum rails and a steel containment, as listed in Table 1.

Figure 4 shows the effects of rail thickness on the critical velocity of railguns. The geometry of containment and all material properties are identical to the baseline case. The increase in the moment of inertia of the rail will enhance the bending stiffness of the rail. Accordingly, the critical velocity increases with the thickness of the rail. However, the critical velocity does not increase linearly; it varies with only a power of 0.25. The stiffness of containment also has strong effects on the critical velocity of the railgun. Figure 5 shows the effect of containment thickness on the critical velocity of the railgun. A thicker containment provides higher stiffness and structural support for the rail. Accordingly, the deflection of containment decreases as it is subjected to magnetic pressure from the rails. Mathematically, it is modeled as the stiffness of foundation,  $k$ , which increases, as illustrated in Figure 5. The effect of the containment stiffness on the critical velocity is not linear either; it varies with a power of 0.75.

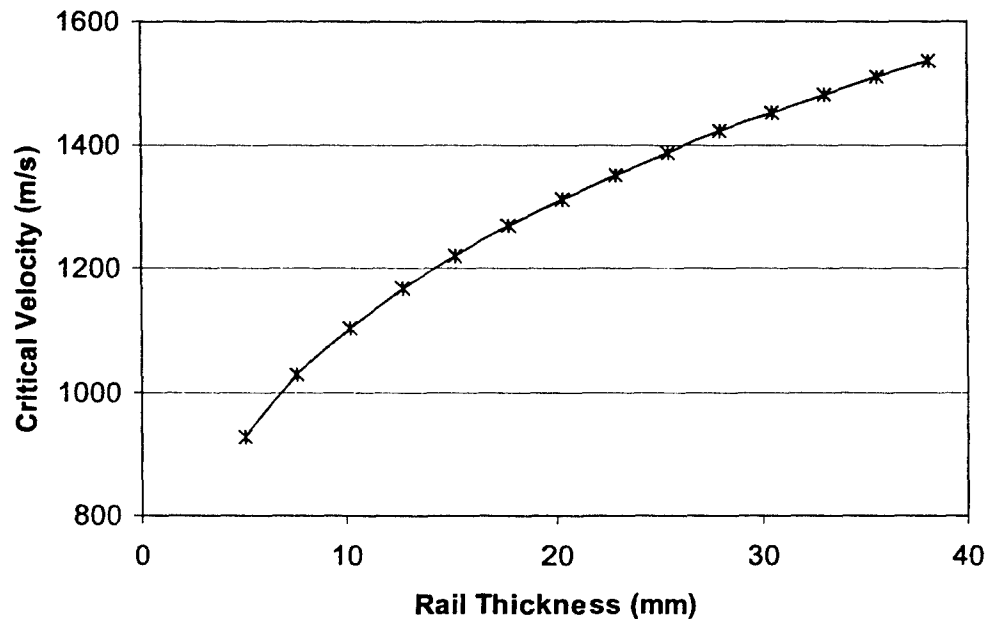


Figure 4. Effects of the rail thickness on the critical velocity.



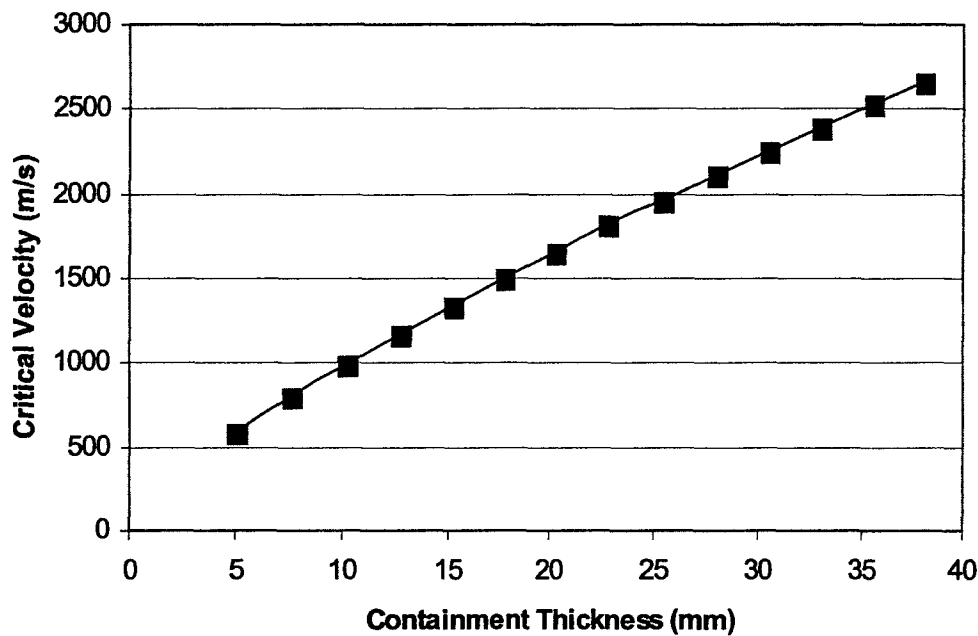


Figure 5. Effects of the containment thickness on the critical velocity.

The effect of rail material properties on the critical velocity is illustrated in Figure 6. A combined effect on the dynamic behavior due to the density and elastic modulus of the rail is illustrated using some potential material choices. The baseline case is 7075 aluminum alloys. Three different conductor materials are examined. GIGAS 24 is an advanced aluminum alloy with a higher modulus of 88.25 GPa (12.8 Msi). The density is about the same as the 7075 aluminum. Accordingly, a higher critical velocity is obtained due to the increase of modulus.

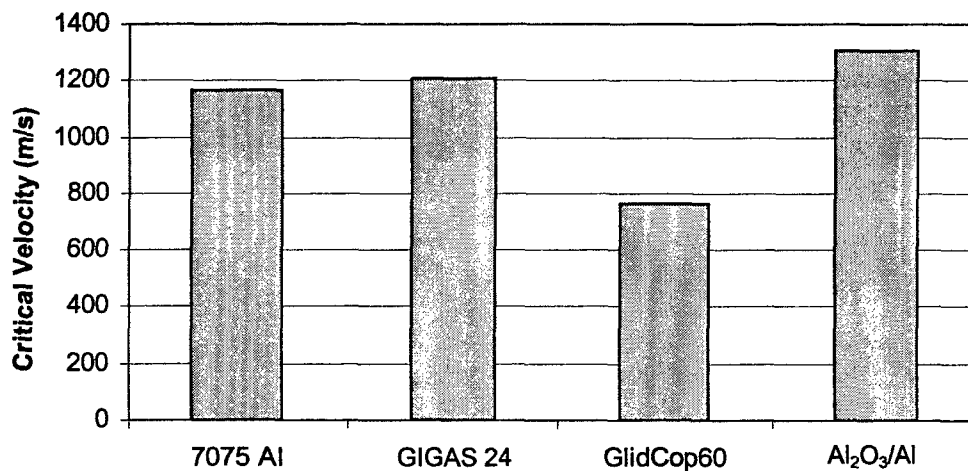


Figure 6. Effects of the rail material properties on the critical velocity.

Glidcop is aluminum oxide dispersion-strengthened copper. It has a high modulus of 172.4 GPa and a high density of 8900 kg/m<sup>3</sup> material. The critical velocity turns out to be lower than the 7075 aluminum due to the high density. Finally, aluminum reinforced with aluminum oxide (Al<sub>2</sub>O<sub>3</sub>) fiber (45% volume fraction of fiber content) is used for comparison. The modulus and density of this material are 165.5 GPa (24 Msi) and 3400 kg/m<sup>3</sup>, respectively. The combination of high modulus and low density gives a high-critical velocity.

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#### **4. Conclusions**

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The dynamic behavior of an EM barrel can be modeled with reasonable assumptions as a rail sitting on an elastic containment. The derived solution illustrates effects of important design parameters and material properties on the critical velocity of a barrel, which can be applied for barrel design under dynamic conditions. A high magnitude of cyclic stress can occur that might cause damage in the rail, accelerate the growth of defect, and eventually shorten the rail life significantly. The dynamic phenomenon is particularly crucial if gun barrels are designed to be a lightweight and fieldable system with hypervelocity capability. The developed model provides a meaningful tool for barrel design that accounts for dynamic response due to a moving projectile.

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